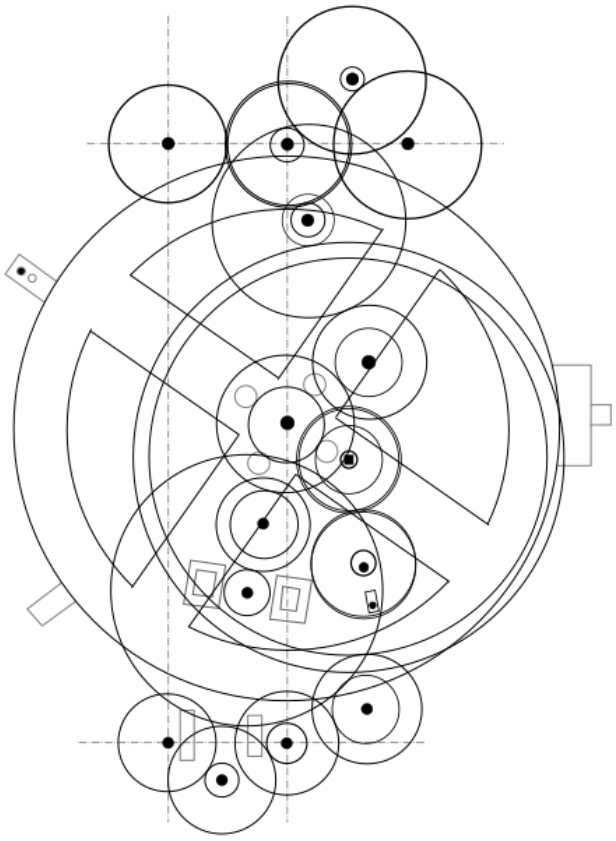

Tampa Campus

College of Arts & Sciences

Department of Mathematics & Statistics

The Early History of Mathematics
Fall 2024
MHF 4403-001; CRN 92435; 3 hours
Class times: TR 12:30 - 1:45 pm
Class meets in CMC013 ... unless otherwise directed.

Text: W. P. Berlinghoff & F. Q. Gouvêa, *Math through the ages: A Gentle History for Teachers and Others* (Oxton / MAA, 2nd ed., 2015 or Dover's Illustrated Edition; Dover is cheaper)

Teacher: Gregory McColm
email: mccolm@usf.edu
Office Hours: TBD
Office: CMC 342/338

This syllabus is subject to change.

Why Take This Course (Besides Because You Have To)?

All around the world, girls and boys have to learn mathematics. School attendance is mandatory, and so is mathematics. Education may be only a recently acquired right - the United States and most of Europe didn't start making children go to school until late in the Nineteenth century, if then - but schools have taught mathematics for thousands of years.

Why? Schools cost money, so there must be a reason why wealthy parents, nobles, corporations, and governments would go to the expense and bother of teaching kids arithmetic and geometry. And mathematics is notoriously frustrating and requires an unnatural and unforgiving precision: for thousands of years, students have been asking teachers why they have to learn mathematics. And schools that were founded by students who wanted to become merchants, lawyers, and politicians, they had math classes, too. Why?

That is what this course is all about. Who were all these people who did mathematics? Why did they do it? And why should we continue to do it? We may have iPads, jet airplanes, antibiotics, cruise missiles, and now AI, but they were people just like us, with the same hopes, fears, ideals, and obsessions. And like us, they did mathematics.

The Catalog and the Course

This course is described in the catalog:

> *A study of the history and development of mathematics and its cultural impact from the formation of number systems to the Renaissance.*

It is one of two courses, the second of which is MHF 4406 / 5405 *History of Modern Mathematics*, which covers the history of mathematics from the Renaissance on. This latter course is not offered much, so we will not stop at the Renaissance but also cover the development of calculus and statistics and machines (including, of course, computers) and some other major developments in the modern era.

The catalog recommends as textbooks V. Katz's *A History of Mathematics: An Introduction*[1] and D. Burton's *The History of Mathematics: An Introduction*[2], both of which are decent books, but you will not need them in this course. Instead, we will use W. P. Berlinghoff & F. Q. Gouvêa's *Math through the ages: A Gentle History for Teachers and Others*; you should get the 2nd edition[3]. All other materials for the course will be free, either posted on Canvas or linked to the web or in the library.

The catalog also states that MAC 2312 *Calculus II* is the sole prerequisite, although MAC 2281 *Engineering Calculus II* would be fine (if you have any problems enrolling with MAC 2281, see me). MAC 2242 *Life Science Calculus II* should be okay - but see me first.

The Content of the Course

A single semester on mathematics from the late Early Stone (Paleolithic) Age to the present is only going to visit a few high points. The course is divided into week-long modules, and very briefly looks like this:

- We start with the basic issues of the course - mathematics, history, human nature, education, etc. - and mathematics before Greece. The most important points are (a) a lot of things we don't think of as being mathematical actually are, and (b) history is about

[1] This book is much larger than the one we are using and goes much deeper into technical detail: it is more of a book on how mathematics was done back then rather than a history book. If you want to see how mathematics was done back then, this is an excellent resource. But have a big jar of elbow grease handy.
[2] This book goes deeper into technical detail and focuses on mathematics developed in ancient and modern Europe.
[3] And the second edition is now available from Dover Books, for $20 including shipping and tax. The primary difference between the Dover edition and the MAA/AMS version is $60, and the MAA/AMS version has all the exercises. But I will give you the exercises I want you to try, so you can just buy the Dover edition. And there may be even cheaper used copies floating around.

human beings, so (c) this course is about human beings doing mathematics - and what they did with that mathematics.

- A few weeks on Greece. The Greeks didn't invent mathematics, but they transformed mathematics into the enterprise it is today. We will explore some of the major aspects of mathematics developed by the Greeks - and the Greek views (plural!) of mathematics - and what they did with the stuff.
- A few weeks on Asia: India, China, and the Middle East. It was the medieval Middle East that organized the mathematics curriculum we have today.
- The rest on Europe and the modern era, including calculus, statistics - and some mathematics we just take for granted.

Of course, a lot of mathematics was developed in sub-Saharan Africa, pre-1492 America, and pre-1492 Austronesia, but that effort had little influence on what is taught today in classrooms around the world. (As we shall see, most of the mathematics taught in modern classrooms was developed from what was available in Mediterranean academies during the Renaissance - and that came from the corridor that runs from Gibraltar to Japan.) Worse, documentary evidence from sub-Saharan Africa, pre-1492 America, and pre-1492 Austronesia has mostly disappeared. Details are in the Schedule, which is subject to change as developments develop.

Objectives

Unlike most mathematics courses, which are supposed to impart certain problem-solving skills while conveying certain knowledge about the subject that those skills address, this course is supposed to convey what mathematics has meant to people - especially the people who developed, used, and taught it - and the part that mathematics played in transforming our society from a panoply of scattered tribes to a technological colossus dominating the planet.

After you have taken the course, you will have:

- An outline of the history of mathematics as a subject, including some major fields covered in elementary and secondary math classes.
- Awareness of some of the influences and sources of mathematics - including things we often don't think of as mathematics.
- Awareness of some of the major applications of mathematics to science and technology, and how those applications - and the desire for such applications - influenced the development of mathematics. And vice versa.
- Awareness of who developed mathematics, who used it, who taught it, who learned it, and why.
- Familiarity with some of the major issues in mathematics, the philosophy of mathematics, and mathematics education.

At the end of this course, you will be able to give an account on any of these points to a colleague, a layman, or a student of your own. The primary goal is to have the answers to the following questions:

- *To students who ask, Why do I have to learn this stuff?*

- *For teachers, too yourself when you ask, Why am I teaching this stuff?*
- *To practicing mathematicians, Why am I doing this stuff?*

But there is another set of objectives for this course - which is related to the subject of this course. As we will find early in this course, schools were started when the newly organized city-states realized that they needed civil servants, engineers, and other literate professionals - including teachers. Nowadays, that includes reading and writing memoranda, reports, bulletins, etc. And artificial intelligence has entered the scene. So there are additional objectives.

- Selecting a research topic, and collecting and organizing material using both traditional tools (in the library), and modern tools (internet sources, Large Language Models - Artificial Intelligence). This includes evaluating the reliability of sources.
- Composing a report that people are likely to read or a presentation people are likely to pay attention to, again using a variety of tools.

How the Course Works

The material is divided into weekly modules, and each module looks like this:

- For each class session, there is a “reading” (often a "sketch" from the text) and a video. Videos, readings, and sketches are in the textbook, posted on Canvas, or linked from the Schedule page. You should read the reading and watch the video *before* coming to class.
- Each class session, students will meet as a group (*attendance is required, and absences must be excused*) and fill out a worksheet based on the reading, video, and the class discussion. The worksheets must be completed on the day class meets.
- Each week, there will be a homework assignment for that week.

Attendance. Absent students will not get credit for their group worksheets for that day, and there are no makeups for the worksheets *unless* the absence is excused.

There will be a term paper and a presentation. Each will be on *a mathematical topic chosen by you and of interest to you*.

- The term paper will be about two thousand words, with no particular requirement on the number of sources, written in the format acceptable to English composition courses. Each week during the first half of the course, you will submit a progress report.
- The presentation will be about twenty minutes, with Q&A at the end. The presentations will be on the day of the final (in lieu of the final). Each week during the second half of the course, you will submit a progress report. The presentation will be recorded for grading purposes.

Academic integrity and other standards will be described in the rubrics posted online.

Deadlines are in the Schedule.

About grades. The rubrics for grading stuff are in the *Handout on Papers*, but here are the basics. Each item is graded on a scale of 0 to 10. The course will have the usual A = 90% & up, B = 80% & up, C = 70% & up, D = 60% & up. The breakdown is:

In-class worksheets	20%
Weekly Homeworks	20%
Term Paper (including progress reports)	30%
Presentation (including progress reports)	30%
Total	100%

Scores are posted on Canvas. I grade all other work manually and return the graded work.

Incompletes. Incompletes are often never made up, so they are strongly discouraged. However, if an Incomplete is absolutely, positively necessary due to illness, alien invasion, vampires, etc., once the student has recovered the student and I will fill out a form specifying what work remains to be done and by what time it should be done; university policy is that work for an Incomplete for this course ***must*** be completed by the end of Summer Session B in 2025.

Other Policies

The following are class policies:

- **You are responsible for every announcement posted on Canvas.**
- **Attendance is required.**
    - If a student knows that (s)he will miss a class, (s)he must tell me in advance. The USF Undergraduate Catalog says: "Acceptable reasons for scheduled absences include observation of religious holy days, court-imposed legal obligations (e.g., jury duty and subpoenas), special requirements of other courses and university-sponsored events (e.g., performances, athletic events, judging trips), and requirements of military service. Employment schedules, athletic training and practice schedules, and personal appointments [e.g. family reunions and vacations] are not valid reasons for scheduled absences." Absences for acceptable reasons will be excused.
    - If a student misses a class because something came up, the student must contact me as soon as possible if it is to be excused. The USF Undergraduate Catalog says: "Unscheduled absences involve unforeseen emergencies such as illness, injury, hospitalization, deaths in the immediate family, consequences of severe weather, and other crises."
- **Gizmos are tolerated with restrictions.**
    - During synchronous sessions, cell phones should be turned off or set on vibrate, and no machines are permitted to be on during class except those used in class, e.g., gizmos for taking notes. However, studies have indicated that not only do gadgets distract students using them, they also distract *other* students[4], so they are not encouraged unless there is a reason for using them.

[4] Something to remember when *you* are teaching a class.

The following is university policy.

- **Early Notification Requirement for Observed Religious Days.** Students who anticipate the necessity of being absent from class due to the observation of a major religious observance must provide notice of the date(s) to the instructor, in writing, at the beginning of the term (see the Early Notification Requirement for Observed Religious Days).
- **Academic Integrity of Students.** Instances of academic dishonesty including but not limited to: cheating, plagiarism, fabrication, forgery, complicity and computer misuse will not be tolerated. For further information about university policies for academic dishonesty, go to the USF page on Academic Integrity for Students.
- **Disruption of the Academic Process.** Please do not hold conversations, either with your classmates or your cell phones, during the discussion sessions. For further information about university policies for disruption of the academic process, go to the USF page on Disruption of (the) Academic Process.
- **Title IX.** The university's lawyers want this statement to be in the syllabus: *Title IX provides federal protections for discrimination based on sex, which includes discrimination based on pregnancy, sexual harassment, and interpersonal violence. In an effort to provide support and equal access,* ***USF has designated all faculty (TA, Adjunct, etc.) as Responsible Employees, who are required to report any disclosures of sexual harassment, sexual violence, relationship violence or stalking.*** *The Title IX Office makes every effort, when safe to do so, to reach out and provide resources and accommodations, and to discuss possible options for resolution. Anyone wishing to make a Title IX report or seeking accommodations may do so online, in person, via phone, or email to the Title IX Office. For information about Title IX or for a full list of resources please visit: https://www.usf.edu/title-ix/gethelp/resources.aspx. If you are unsure what to do, please contact Victim Advocacy – a confidential resource that can review all your options – at 813-974-5756 or va@admin.usf.edu.*

  The Center for Victim Advocacy and Violence Prevention, the Counseling Center and Student Health Services are confidential resources where you can talk about such situations and receive assistance without the incident being reported.

  - *Center for Victim Advocacy and Violence Prevention:*
    1. *(813) 974-5757*
    2. ***https://www.usf.edu/student-affairs/victim-advocacy/***
  - *Counseling Center*
    1. *(813) 974-2831*
    2. ***https://www.usf.edu/student-affairs/counseling-center/***
  - *Student Health Services*
    1. *(813) 974-2331*
    2. ***https://www.usf.edu/student-affairs/student-health-services/***
- **Students with Disabilities.** Students in need of academic accommodations for a disability should consult with the office of *Students Accessibility Services* (SVC 1133, 813 974-4309) for assistance. The student must bring a current Memorandum of Accommodations from the SDS office; this is a prerequisite for receiving accommodations. Students are required to give reasonable notice (typically five to ten business days) prior to requesting an accommodation. Students may request accommodations at any time throughout the semester. However, as accommodations are

not retroactive, any grades earned before a student requests accommodations will typically stand. Note: there are no exams in this class. It is strongly recommended that students with disabilities make arrangements at the beginning of the semester; see https://www.usf.edu/student-affairs/student-accessibility/ for details.

- **University Emergency Policy.** In the event of an emergency, it may be necessary for USF to suspend normal operations. During this time, USF may opt to continue delivery of instruction through methods that include but are not limited to: Canvas, Teams, Skype, and email messaging and/or an alternate schedule. It's the responsibility of the student to monitor Canvas site for each class for course specific communication, and the main USF, College, and department websites, emails, and MoBull messages for important general information.
- **COVID-19.** We are (still! yes we are!) in the midst of a pandemic that may be receding - but this virus is the comeback kid. We are scheduled to meet in class, but we may be moved online, in which case we will meet synchronously during class times. While we meeting in class, students are strongly recommended to wear masks and practice social distancing: two things to remember are:
    - Even among unvaccinated people, about half the cases are non-symptomatic but still contagious. So even if you don't feel ill, if you are infected you can infect someone else.
    - Masks primarily protect the people around you (that's why surgeons wear masks while operating: to protect the patients). Since vaccinated people can get low-grade or non-symptomatic COVID-19 infections, even vaccinated people are urged to wear masks indoors *in order to protect other people*.
    - Some people cannot get vaccinations for medical reasons, like cancer or autoimmune disease. These are among the people we want to protect by wearing masks.

  Students who have not received a vaccination are *strongly urged* to get one: almost all hospitalizations of COVID-19 patients are of people who did *not* get vaccinated.
    - And a recent advisory from the USF COVID-19 Team: Students should not attend class if they are ill, particularly if they have fever and/or gastrointestinal symptoms and/or respiratory symptoms such as a sneezing, runny nose, sore throat or coughing. Students experiencing any of these symptoms should contact Student Health Services immediately (813-974-2331) for appropriate medical guidance and to obtain a verification of care letter. **To be approved for missed classes, late assignments or missed examinations a verification of care letter must be presented by the student to the faculty member upon return to class.**

  Meanwhile, since we are dealing with surges, students should be prepared to move online at a moment's notice.

There is also state policy.

- **Class recordings.** A new law - Florida House Bill 233 - has gotten a lot of press because, among other things, it empowers students to record class lectures without asking permission. But there is a lot of fine print, and the USF provost's office has sent out the following bullets.
    - Students with disabilities will continue to have appropriate accommodations for recordings as established by SAS.

- Students may record class lectures which will be considered instructor-delivered academic content. No recordings of other students, class participation, or discussion will be permitted.
- Students do not need advance permission or to provide notice to record. But, students must monitor their recording so that it does not include participation by other students.
- Students may not publish (post or share) the recordings except as provided by statute. In the event permission is requested to publish (post or share), the student's request and instructor's consent must be in writing.
- Students and instructors are responsible for compliance with the statute and related potential sanctions. Misuse of the recordings may result in referrals as possible violations of the student code of conduct or considered academic disruptions.

What the provost's office did not mention is that these recordings can fall afoul of copyright laws if they are shared.

- **Academic freedom.** Another new law - Florida House Bill 7 - says that faculty are not supposed to take positions on a list of topics that legislators are concerned about. It turns out that I do have opinions on some topics (*math is great* and *history is how we find out what human nature really is*), but legislators don't care about those. At any rate, this class is not about forming "correct" opinions about things - we will see that lots of major historical figures disagreed with each other on major issues - it is about how mathematics plays a part in human affairs … and in our own lives.
  Nevertheless, the university's lawyers would like this statement to be in the syllabus: *It is fundamental to the University of South Florida's mission to support an environment where divergent ideas, theories, and philosophies can be openly exchanged and critically evaluated. Consistent with these principles, this course may involve discussion of ideas that you find uncomfortable, disagreeable, or even offensive.* For an example, see the Trigger Alert below.

Trigger Alert

This is a course about mathematics and the human condition, which means we are going to be studying human beings. Human beings can be courageous or cowardly, farsighted or foolish, kind or cruel, virtuous or vicious, and this is true of mathematicians as well. They span the spectrum.

For example, during World War II, Alfred Rényi was hiding in Budapest when his parents were trapped in the ghetto. Rényi got hold of a Nazi uniform and marched them out as if they were his prisoners, right past the guards. At the other extreme, Ludwig Bieberbach - whose work on geometry is central to my own research - was the most pre-eminent mathematician in the Nazi Party, and he was an enthusiastic party member who persecuted Jewish colleagues during the war, and an unrepentant Nazi afterwards.

Alfred Rényi worked in combinatorics and was concerned about military applications of science and technology. His most famous quote is: "A mathematician is a machine for turning coffee into theorems."

Ludwig Bieberbach's early work in geometry - partially solving David Hilbert's 18th problem - is central to my own research. But he is best known for his work in calculus with imaginary numbers - and for his "German mathematics."

In most mathematics courses, this kind of thing doesn't come up much. But this course is about how mathematics and human beings have interacted, and while we would like great mathematicians to be good people - if only because then they could be good role models for students - this isn't always the case.

And that is only a small part of this trigger alert. Imagine that morality and ethics were something societies had to learn, slowly and painfully. Imagine that we are where we are now because of what we've learned over the past few thousand years (at least). What were we like back then - before we learned how to behave as well as we do now? "...as well as we do now?" Harvard psychologist Steven Pinker stirred up a storm by claiming that per capita, violence has dramatically declined over the last few thousand years. This claim is quite controversial, but it is true that violence has dramatically declined in Europe since the Thirteenth and Fourteenth centuries (an episode that will loom large in this course). In fact, *per capita*, the Twentieth century, with its world wars and genocides, was a lot less violent than the Thirteenth and Fourteenth.

We don't notice this if we take movies like *Lord of the Rings* or *Game of Thrones* to be the past: such films are prettified in order not to shock the audience. But this is a history course, and an important part of this course is how life got better over time, *because mathematics played a critical part in making life better* - although, as we shall see, mathematics occasionally played a less laudatory role. So as much as possible, we will try to study the past as it actually was, not as we would prefer it to have been.